**UDC 535.232**

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# AMPLITUDE AND INTENSITY DISTRIBUTION AROUND A FOCAL POINT OF ANY OPTICAL SYSTEM

***Abstract.*** The paper describes optimal methodology for calculation amplitude and intensity distribution around a focal plane of an optical system

***Key words:*** amplitude, intensity, distribution, optical system

## INTRODUCTION

Focusing of light is an exciting physical phenomenon. With the development of engineering and information technologies, it became possible to use more complicated mathematical apparatus for calculation of an amplitude and an intensity of a spherical wave without making many simplifications or approximations.

## AMPLITUDE DISTRIBUTION

According to Huygens-Fresnel principle, each point on a spherical wave front plays a role of a secondary point source [1, 2]. The resulted complex amplitude in any point around a focal point is proportional to the superposition with interference of all waves coming from these secondary point sources. It is possible to find the optical path length of the beam coming from the point A at the wave front to the point B near the focal point F [1]. Then we write the following expression for calculation of the complex amplitude in the point B made by the point source A [3]:

$$
\begin{aligned}
E_P(s,r,t) &= E_O \cdot \frac{\exp\left\{ j2\pi\left[\frac{1}{\lambda}\cdot n\cdot L(s,r) - \frac{t}{T}\right]\right\}}{L(s,r)} \approx E_O \cdot \frac{\exp\left\{ j2\pi\left[\frac{n}{\lambda}\cdot L_A(s,r) - \frac{t}{T}\right]\right\}}{L_A(s,r)} = \\
&= E_O \cdot \frac{\exp\left\{ j2\pi\left[\frac{n}{\lambda}\cdot (R + s\bullet r) - \frac{t}{T}\right]\right\}}{(R + s\bullet r)} \approx E_O \cdot \frac{\exp\left( j2\pi\cdot\frac{n}{\lambda}\cdot R\right)\cdot \exp\left\{ j2\pi\left[\frac{n}{\lambda}\cdot (s\bullet r) - \frac{t}{T}\right]\right\}}{R} = \\
&= E_O \cdot C_R \cdot \frac{\exp\left[ j2\pi\cdot\frac{n}{\lambda}\cdot (s\bullet r)\right]}{\exp\left( j2\pi\frac{t}{T}\right)}
\end{aligned}
\tag{1}
$$

where $E_P(s, r,t)$ – the complex amplitude of the wave coming from the point A to the point B; $n\cdot L(s,r)$,$n\cdot L_A(s, r)$ – the precise and approximated values of the optical path length; $s = (s_X, s_Z)$ – this vector defines the Cartesian coordinates of the point A relatively to the focal point F, its length is equal to 1: $|s| = 1$, its coordinates are equal to $s_X=\sin(\phi)$ and $s_Z=\cos(\phi)$; $\phi$ – the angular coordinate of the vector s relatively the optical axis z ;

r = (x,z) – this vector defines the coordinates of the point B relatively to the focal point F, this vector has the Cartesian linear coordinates (x, z), and according to the assumptions $\lambda \ll R$, $|r| \ll R$ , $(r \bullet s) \ll R$; $E_O$ – the initial amplitude of the spherical wave coming from the point A; R – the radius of the spherical wave front coming from the aperture stop; • – the symbol of the scalar product of two vectors; j, $C_R$ – the constants:

$$j^2 = -1; \qquad C_R = \frac{1}{R} \cdot \exp\left\{ j2\pi \cdot \frac{n}{\lambda} \cdot R \right\}$$

The expression (1) helps to calculate the distribution of complex amplitudes in the object space around the focal point F. According to Huygens-Fresnel principle, each complex amplitude in the point (x, z) is a result of the superposition with interference of secondary spherical waves coming from points at the wave front (1). To get this distribution it has to calculate the following integral for all points (x, z) (Fig.1) [3]:

$$\begin{aligned} E(r) &= \int_{-\sigma}^{+\sigma} p\left[s(\varphi)\right] \cdot c\left[s(\varphi)\right] \cdot E_p\left[s(\varphi), r\right] d\varphi \approx \\ &\approx \int_{-\sigma}^{+\sigma} p\left[s(\varphi)\right] \cdot c\left[s(\varphi)\right] \cdot E_O \cdot C_R \cdot \frac{\exp\left\{ j2\pi \cdot \frac{n}{\lambda} \cdot \left[ s(\varphi) \bullet r \right] \right\}}{\exp\left( j2\pi \frac{t}{T} \right)} d\varphi = E(x,z) = \\ &= E_O \cdot \frac{C_R}{\exp\left( j2\pi \frac{t}{T} \right)} \cdot \int_{-\sigma}^{+\sigma} p(\varphi) \cdot c(\varphi) \cdot \exp\left\{ \frac{j2\pi n}{\lambda} \cdot \left[ x \cdot \sin(\varphi) + z \cdot \cos(\varphi) \right] \right\} d\varphi \end{aligned} \qquad (2)$$

where E(r) = E(x, z) – the distribution of complex amplitudes around the focal point as the function of the coordinates (x,z) ; p(s) – the complex pupil function that describes amplitude and phase modulation introduced by an OS, if it is no modulation then p(s)=1; c(s) – the weight function that takes into account the beam inclination.

**INTENSITY DISTRIBUTION**

Relevant time averaged intensity may also be found:

$$I(x,z) = C_I \cdot \left| E(x,z) \right|^2 = C_I \cdot E(x,z) \cdot E^*(x,z)$$

where I(x, z) – the distribution of intensity around the focal point as the function of the coordinates (x,z) ; $C_I$ – the constant [1, 2].

To make a good illustration of this process in two-dimensional space there have been developed the mathematical apparatus and free software package. It allows placing the definite number of points on a circular curve. Each point acts as a point source and it emits a spherical wave with a given wavelength. Application of Huygens-Fresnel principle makes possible to calculate the distributions of summarized amplitude and intensity in any point round these point sources. As a result, we can observe the two dimensional distributions round a focal point. It helps to students and schoolchildren to understand wave optics (Fig. 1).

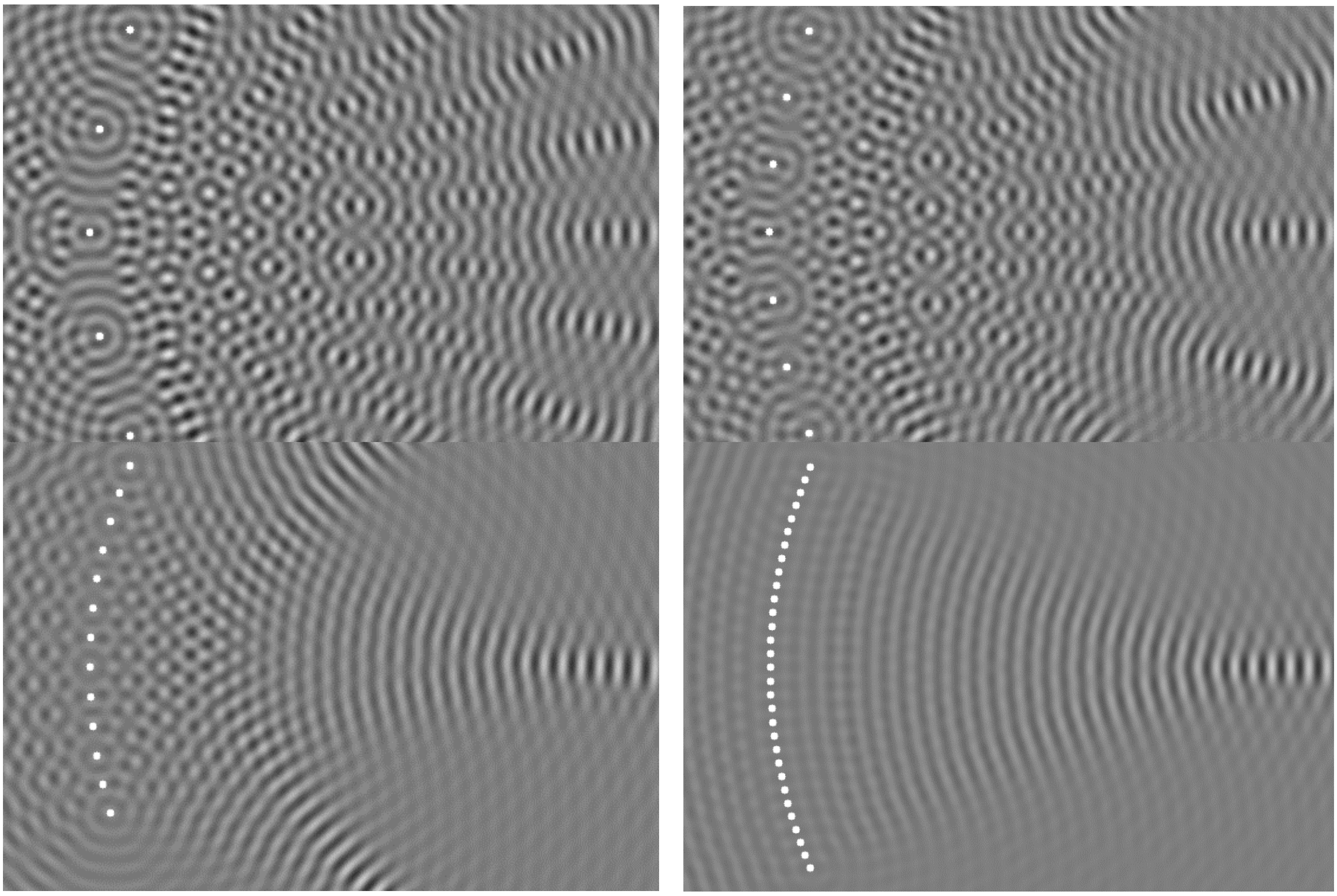

Fig 1. Two dimensional distribution of amplitude generated for the cases with various number of point sources

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